Paper prepared for presentation at the Annual Convention of the International Studies Association, Montreal, Quebec, Canada, March 16–19, 2011.

Pandemonium in Silico: Individual Radicalization for Agent-Based Modeling

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Abstract. How do individuals become radicalized, turning into terrorists, insurgents, violent actors. Computational agent-based models of irregular warfare, internal war, domestic political violence, and related conflicts require violent agents capable of carrying out attacks. Rather than introducing such agents as an exogenous process, as a *Deus ex machina*, this paper presents an agent-based model where radicalization is generated as an emergent phenomenon from within a population of individuals. The model (tentatively called "MASON RadicalAgent") is based on a new process-based theory of individual radicalization and is implemented in the MASON simulation system. Our paper describes the underlying theory, model structure, and some preliminary results intended for demonstration. This modeling effort is part of a broader project for modeling conflict in complex polities by combining computational simulations and network models.

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How do individuals become radicalized, turning into terrorists, insurgents, or agents of related kinds of violence? Computational agent-based models of irregular warfare, internal war, domestic political violence, and related conflicts require violent agents capable of carrying out attacks against their targets. Rather than introducing such agents as an exogenous process, through some *Deus ex machina*, this paper presents an agent-based model (ABM) where radicalization is generated as an emergent phenomenon from within a broader population of individuals belonging to society. The model (tentatively called "MASON RadicalAgent") is based on a new process-based theory of individual radicalization that is implemented in the MASON simulation system. Our paper describes the underlying theory, model structure (specification and implementation), and some preliminary demonstration results for illustrative purposes. This modeling effort is part of a broader project for modeling conflict in complex polities by combining computational simulations and network models.

This paper contains five sections. In the first, we provide motivation and background. The second section describes how the MASON RadicalAgent model was developed in terms of specification and implementation. The third section presents a set of illustrative results. The fourth section presents a discussion of the model in terms of features, extensions, and future directions.

1. Introduction

1.1 Motivation

In real societies—past or contemporary—radicalized individuals are not born as radicals, nor do they just "come out of the blue." Instead, radicalized individuals become that way through some

transformation process that takes them "from cradle to terror."—and often "to grave" as well. However, in most social or socio-natural ABMs where conflict or violence occur, the agents of violence are usually predetermined in the simulation (e.g., Epstein, 2002) or, alternatively, they may turn violent through some exogenous process. Some exceptions exist, as we discuss below, but most ABMs do not model radicalization endogenously.

The main challenge addressed in this paper is the endogenous production or emergence of radical violent agents as an emergent phenomenon generated within a broader population of agents i.e., how to generate radical agents without hard-wiring them as individual extremists. There are basically two ways to address this challenge. The first is to draw on relevant social science (namely social psychology, behavioral science, and related social science of radicalization) to obtain the proper ideas for modeling radicalization. The second way is to rely on arbitrary algorithmic procedures that yield comparable results (pseudo-generative mechanisms, as these may be called), such as random assignation or other heuristic, non-theoretical procedures. Here we select the former strategy as a development in computational social science.

1.2 Background: Relevant extant literature

Insurgents, guerrilla fighters, rebels, terrorists, suicide bombers, violent anarchists, and similar kinds of extremist contentious actors have been implemented in social science simulation models since the early days of discrete dynamical systems (e.g., Ruloff's (1975) DYNAMO model of guerrilla warfare in Afghanistan during the Soviet occupation). Other dynamical systems models based on similar aggregate frameworks include Akcam and Asal (2005), Madnick and Siegel (2007), and Wakeland and Medina (2010), among others.

In the area of agent-based models (ABMs), including those with an explicit territorial or geospatial component (spatial ABMs), contentious actors have been included in models since this area of computational social science began focusing attention on dynamics of political instability, regime fragmentation and change, civil war, secession, insurgency, and related phenomena. For example, guerrilla insurgency was implemented in the Iruba modeling project (Doran, 2005), which models a 32-province country afflicted by insurgency and related violent contentious politics between government forces and armed opposition. However, in the Iruba model radicalized agents exhibiting violent opposition are either created at initialization or they are "recruited"—i.e., they are not generated as emergent social process. Similarly, in Epstein's (2002) civil violence models, individual agents behaving violently against government forces are not generated by any social or psychological mechanism. Rather, agents rebel based on grievance that is exogenously produced by random uniform values lacking systematic justification in social theory. The same is true of other models, such as Cioffi and Rouleau's (2010) more recent RebeLand model, where agents turn radical when they become dissatisfied with their situation and as a result rebel against government; not because they undergo any psychological or cognitive transformation. This general theoretical deficiency in extant models of political violence does not detract from their contribution to a better understanding of the complex phenomena involved; but it does motivate development of models grounded in richer theoretical foundations

In sum, to date there has not been an ABM where the phenomenon of individual radicalization is modeled as a psychological and cognitive process of transformation supported by viable social theory. Such a model would provide a number of advantages, such as a more systematic specification and implementation of radicalization—what generates insurgents, terrorists, guerrilla fighters, and other violent actors—in terms of known cognitive and psychological mechanisms.

2. Method: Model Development

In this section we describe the procedure used to develop the RadicalAgent model in terms of specification, implementation, verification, and validation. We follow Sargent's (2004) and Cioffi's (2010a) criteria for verification and validation.

2.1 Model development: Specification and implementation

The model was specified using Cioffi's (2010b) formal theory of individual radicalization and implemented in the MASON system (Luke et al., 2005).

2.1.1 Model specification

The main specification objective for RadicalAgent was to create a model capable of occasionally generating radicalized agents based on a broader population of heterogeneous, bounded-rational agents (common people) undergoing situational changes in their lives. As summarized in Figure 1 and detailed in elsewhere (Cioffi, 2010b), individual radicalization is a process of cognitive and psychological transformation, not something individuals are born with or an occurrence "out of the blue." The process of individual radicalization consists of three requirements (logically necessary conditions): *Traumatic grievance, extremist indoctrination*, and *loss of killing inhibition*. Roughly speaking, the first provides the motive for conducting violent attacks. The second provides cognitive support. The third disables murderous inhibitions humans have evolved as social members of a community.

In turn, each of these three events has a more detailed, fine-grained causal structure, down to so-called "leaf events" that mark the resolution of the tree. The overall causal mechanism is specified in terms of Boolean AND and OR connectives, corresponding to causal conjunctions and disjunctions, respectively. Formally, the occurrence of individual radicalization is specified in terms of a structure function expressed in terms of causal events linked by logic Boolean connectives. The emphasis here is on the use of the tree in Figure 1 for building the RadicalAgent model.

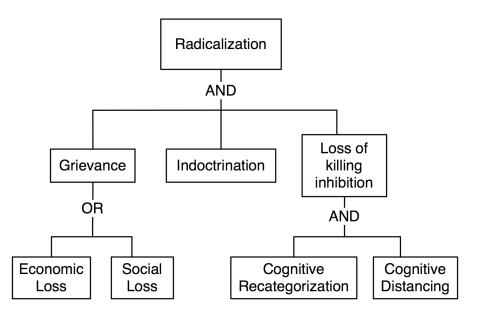


Figure 1. Backward-logic, conditional event tree for individual radicalization. The top, main event of interest is given by individual radicalization occurring as a "root event." The root event will occur when the events below it have occurred, which depend on the events below them. The overall tree represents a theory of individual radicalization based on three requisite conditions linked by a first-level Boolean AND connective: traumatic grievance, extremist indoctrination, and loss of killing inhibition. Additional causal levels are specified by other AND and OR connectives until reaching "leaf events" that mark the resolution of the tree. *Source*: Adapted from Cioffi (2010b).

Model specification consisted of several steps. First, we identified the leaf events in the success tree for individual radicalization. These events include, for instance:

- "Economic Loss:" An individual is grieved by significant loss in wealth.
- "Social Loss:" An individual is grieved by a decline in social status.
- "Cognitive Recategorization:" Out-group categorization. An individual is classified as a member of a different group, thereby losing membership in a prior group.
- Other leaf events. Additional leaf events are specified in Cioffi (2010b).

Second, we identified a set of events that actually occur as part of a simulation process. (The simulation model in which RadicalAgent is inserted may be an existing model or some other ABM.) Examples of this other set of events include:

- Economic transactions. Sales of goods, transfers of money, returns on investments. Some of these can cause economic loss.
- Political events. Campaign events, elections, appointments. Some of these can cause social loss.
- Loss of economic status. An individual undergoes significant losses in the course of work or other economic transactions.
- Loss of social status

- Migration of agents. Movements of population due to push or pull factors, such as conflict, natural disasters, attractive opportunities.
- Other events.

The individual grievance model can be viewed as a System Dynamics model. The level of grievance is occasionally increased by traumatic events and decreases via a decay process that can have various forms: Exponential, linear (constant decay), parabolic (time-since-last-trauma decay), and sigmoidal, each with a characteristic half-life (t such that the level of grievance is half the initial value G₀). Grievance events can be generated stochastically or from a file containing a stream of events. Figure 2 shows examples of these four patterns of decay driven by regularly spaced stochastic shocks of varying intensity. Stochastic generation of grievance events enables simulation of alternative scenarios using various assumptions, although an exponential distribution (Poisson) of time-between-events is common in many social systems and processes (Bartholomew, 2005; Cioffi, 1998: 52, table 2.1).

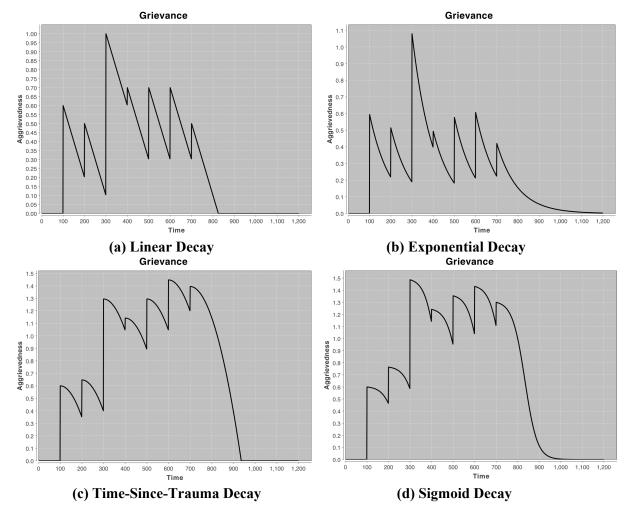


Figure 2. Level of traumatic grievance (y-axis) as a function of time (x-axis), given four different decay functions (linear, exponential, parabolic, and sigmoid) under an identical set of time-distributed shocks. Adapted from Harrison and Cioffi (2010).

Each agent is linked to other agents through a network defined by agent-nodes and a set of relations of kinship and friendship, representing a social structure. Given an individual agent A_i with grievance level G_i , the occurrence of traumatic events generates "echoes" of grievance in other agents, as determined by social structure.

An example of the echo mechanism is shown for four settings in Figure 3, assuming four different (illustrative) social structures. The simplest result is the 4-set, 1 echo case, where an individual agent's traumatic grievance is felt by four local von Neumann neighbors (Figure 3a). In the most complex case, 8-set, 2 echoes, the effects are felt throughout a broader community (Figure 3d).

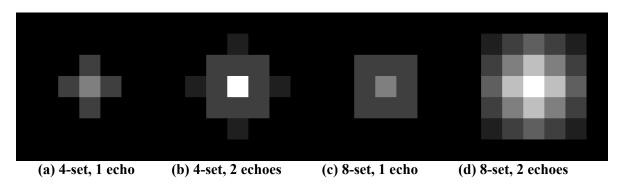


Figure 3. Echo effect of traumatic grievance. In (a) the center agent suffers a traumatic grievance which is echoed to its von Neumann neighbors at 50% off its original intensity. In (b) the event is echoed to the von Neumann neighbors, then echoed again to each of their neighbors, including back to original agent. (c) and (d) show the same process but using the Moore neighborhood instead. Adapted from Harrison and Cioffi (2010).

The second requirement in the process of individual radicalization (Figure 1) is indoctrination into an extremist belief system. Indoctrination was modeled as formation of extremist views in an opinion dynamics model, building on Jager and Amblard (2005). As part of this we replicated the Jager-Amblard model in MASON (Harrison and Cioffi, 2010), successfully reproducing patterns of opinion polarization under a variety of conditions.

Extending the Jager-Amblard model, we also introduced demagogues (extremist preachers) in the social network. Demagogues are akin to signal transmitters of extremist views with unchanging opinions and capacity to instigate further extremism.

The procedure just described resulted in an agent-based model where individual radicalization is generated by endogenous iterations, not hard-wired into the model. Emphasis here is on the more complex, multi-level causality of the grievance component, as opposed to the indoctrination and killing inhibition components of the model in Figure 1.

2.2 Model verification

The model was verified and debugged to ascertain internal validity. Static verification consisted of a code walk-through, ensuring that the main specifications had been properly implemented. Dynamic verification consisted of conducting several parameter sweeps. Calibration was established qualitatively, given the level of abstraction of the model. Additional procedures for model verification and calibration will be used when RadicalAgent is added to other models, such as RiftLand or Poppystan.

2.3 Model validation

The model was validated by two tests. First, a test of face validity showed credible results for radical opinion formation, including comparisons with the earlier Jager-Amblard model, as shown by results in the next section. Second, the skew distribution of opinions was also in conformity with Kellstedt's (2010) findings on the rarity of extremist attitudes within the context of a general population.

2.4 Sample simulation runs

The sample simulation runs reported in the next section were selected for their illustrative value, in terms of showing how the model works and some of the main operational features. The model can also be used for analyzing the differential impact of various types of traumatic grievance events, or different prerequisites of individual radicalization. In this paper the sample simulation results pertain primarily to the effect of traumatic grievance in the life of an individual.

3. Illustrative Results

In this section we present some preliminary test results by way of illustration. We present three types of results that illustrate some of the functionality and behavior of the MASON RadicalAgent model. Other results are available using different parameter setting or by varying the input design.

3.1 Grievance Generation and Decay

Figure 4 shows a traumatic grievance process in a typical run resulting from situational events in the life of an individual (e.g., loss of wealth, loss of social status, etc.), assuming a sigmoidal decay. The temporal spacing of traumatic events in Figure 4 is equal to 100 time units, for illustrative purposes only; we relax this assumption in the next result. (A different decay function generates a somewhat different pattern of crests, depending on the functional form.) The agent begins in a ground state experiencing no grievance at time t = 0. A series of traumatic events

(which are read from a file) affect the agent, causing an increase in the agent's grievance level ("aggrievedness"). The first occurs at time 100 and has intensity 0.6. After that trauma, the agent slowly begins returning to normal but, at time 200, another event occurs. Had it not happened so soon, the agent would have had returned to its "ground state." When a much larger event occurs at time 300 it pushes the agent past its metaphorical "breaking point" at 1.0 and makes the agent susceptible to radicalization.

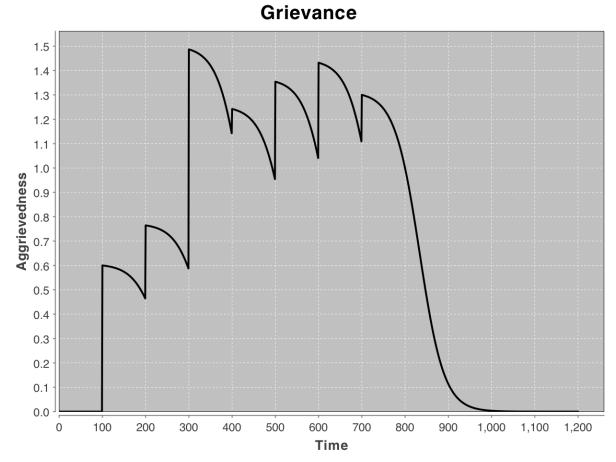


Figure 4. Grievance level of a RadicalAgent in response to a series of traumatic shocks. At time 100 the agent experiences a traumatic event with intensity 0.6. The agent's grievance level increases accordingly, then slowly begins to decrease. At time 200 another small event (intensity 0.3) occurs, and a much larger (intensity 0.9) one hits at time 300. Because events happen in rapid succession, the agent doesn't have time to recover and is pushed past its "breaking point."

The series of traumatic events in the RadicalAgent model (i.e., events timed 100 units apart in Figure 4) can be gathered in different ways: (1) as the output of other models, (2) hand-coded (as in Figure 4), or (3) randomly generated (as in the next result). For the random generation of events, the model has parameters for adjusting the probability of an event occurring during any given step and the size of the event.

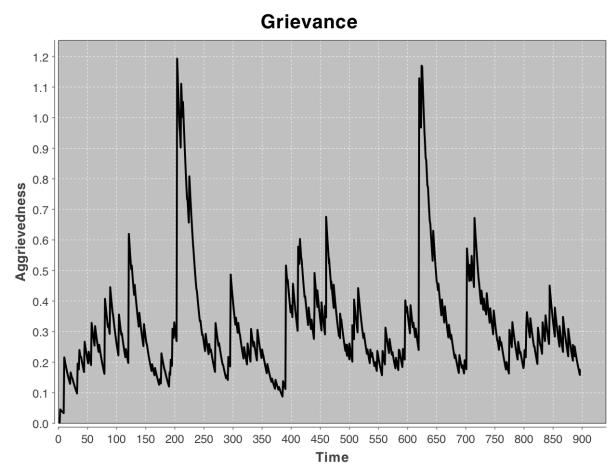


Figure 5. Grievance level of a RadicalAgent in response to a series of randomly generated traumatic shocks. Large shocks around time t = 200 and t = 600 push the agent above the threshold. While above the threshold, the agent is susceptible to radicalization.

Figure 5 shows the results of randomly generated traumatic shocks on the grievance level of an agent. The timing of the shocks are drawn from a Poisson distribution with $\lambda = 5$ and the size of the shocks are drawn from a lognormal distribution with $\mu = 0$, and $\sigma = 1$. The grievance decays exponentially with $\lambda = 0.05$.

3.2 Event Echoes

A traumatic event befalling a person affects not just them, but also that person's family and close friends. To capture this dynamic, the RadicalAgent model has a mechanism whereby events are echoed, in diminished form, from one agent to its neighboring agents. Agents are laid out in a 2D grid (i.e., lattice) and the set of neighbors is defined using either the von Neumann (4-set) or the Moore (8-set) neighborhood.

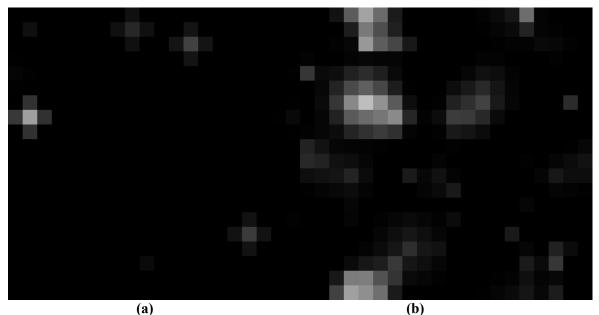


Figure 7. Echo effect in a 20 by 20 grid of agents experiencing randomly generated traumatic events. The color gradient indicates the level of grievance, with black being zero and white indicating the threshold. The events in (a) are echoed once using a von Neumann neighborhood. In (b) the events are echoed twice using the Moore neighborhood.

Figure 7 shows the effect of event echoes in a multi-agent population. Random shocks are generated using a Poisson distribution ($\lambda = 1$) for the timing, and a log-normal distribution ($\mu = 0, \sigma = 1$) for the size of the events. The larger neighborhood effects and multiple echoes shown in 7(b) show how the model parameters can be used to control the level of interdependence between agents.

4. Discussion

In this section we discuss initial results from the model, broader implications, and future research directions.

4.1 MASON RadicalAgent

The results reported in this paper are primarily focused on the first component of the individual radicalization process: experiencing traumatic grievance (see Figure 1), based on the theory (Cioffi, 2010b). The "Indoctrination" component was explored preliminarily in Harrison and Cioffi (2010) and will receive more attention in future work, as will the "Loss of killing inhibition" component. As mentioned earlier, grievance-inducing events (loss of wealth, loss of social status, etc.) occur in many simulation models as part of the "normal" life of agents. In Harrison and Cioffi (2010) we also explored exposure to demagogues and its effect on individual

radicalization as part of the "Indoctrination" condition in Figure 1.

The precise distribution of real social phenomena matters greatly. The temporal distribution for many social events is Poissonian or approximately so. Formally, this means that the probability density function (p.d.f.) of time between occurrences, or durations, follows a simple negative exponential function, $p(t; \lambda) = \lambda \exp(-\lambda t)$. Accordingly, the mean time between events (MTBE) is $\mu = 1/\lambda$. The second set of results (Figure 5) is based on this assumption, making the time intervals between traumatic events more socially realistic than the periodically time shocks in Figure 4.

Figure 5 is also significant because the size distribution for the traumatic shocks is lognormal (i.e., "heavy tailed"), not Poisson or exponential. This second feature adds empirical realism, since "size" variables (wealth, income, territory, area) is more commonly a non-equilibrium distribution such as lognormal or power law. The distinction between these distributions matters in order to generate a process that more closely resembles the experience of individual radicalization in the real world.

Little is known about the cross-cultural variation of individual radicalization characteristics. For example, culturally dependent features, such as susceptibility, exposure rates, or robustness of killing inhibitions require further investigation. The results reported in this paper provide an initial idea of the potential impact of such factors by relating them to parameters explicitly modeled in RadicalAgent. For instance, various decay functions can be used to simulate distinct cultural patterns and compare the effect on radicalization.

4.2 Broader theoretical implications

How does individual radicalization by RadicalAgent compare with previous models? What are some advantages or key innovations? As we pointed out in our literature review, some earlier models of conflict—such as Iruba and RebeLand—have included radicalization. However, this is the first computational ABM where agents radicalize via a theoretically-based process of cognitive transformation. RadicalAgent implements a process-based theory of individual radicalization (Cioffi, 2010b), so the simulation runs are interpretable as "deductions" from the theory, given a set of causal assumptions. *The process theory implemented thus far adds value in so far as earlier models have not been based on a theoretically defensible process*. By the same token, a different theory would yield a different model, which could then be compared with RadicalAgent.

Another implication of having a computational formalization of individual radicalization, in the form of an ABM such as RadicalAgent, is that it provides an interdisciplinary invitation to others working on related fields—e.g., social psychology, behavioral science, criminology. In this way, the model can serve as a bridge between expertise and simulation results, such that the former may contribute toward improving the latter. For instance, not all radicalization experts will agree with the assumptions that underlie the specific theory implemented in RadicalAgent. However, their objections or comments may be amenable to formalization and implementation in code.

4.3 Future research directions

RadicalAgent can be developed in two directions: as a stand-alone model and as part of other models. First, the model is susceptible to a wide range of analyses as a stand-alone model. For instance, we can extend the occurrence of causal events to "Indoctrination" and "Loss of killing inhibition," the other two requisites in the theory of individual radicalization. The result would be a model for the main event of interest (top event in Figure 1) as a function of numerous micro-level events that, when combined, would then generate radicalization. A related extension of the model would add events such as training in weapons and tactics, which would produce terrorist agents when combined with radicalized individuals.

A second viable development—and the original motivation for RadicalAgent—is to use this model as a generator for radical extremists within other agent-based models where we want to have radicalized agents included. In the immediate future we will apply this model to two existing MASON models: (1) Poppystan, a model of Afghanistan developed for the purpose of analyzing a variety of scenarios in modern present-day Afghanistan; and (2) RiftLand a model of East Africa for analyzing complex interactions between society and environmental conditions, including climate change and conflict.

5. Summary

How do individuals become radicalized, becoming terrorists, insurgents, and other agents of violence. Computational agent-based models of irregular warfare, internal war, domestic political violence, and related conflicts require violent agents capable of carrying out attacks. Rather than introducing such agents as an exogenous process, as a *Deus ex machina*, this paper presented an agent-based model where radicalization is generated as an emergent phenomenon from within a population of individuals. The model (tentatively called "MASON RadicalAgent'') is based on a new process-based theory of individual radicalization and implemented in the MASON simulation system. The theory is based on the critical role of three necessary conditions for individual radicalization to occur: experiencing a traumatic grievance, undergoing indoctrination into an extremist belief system, and losing killing inhibition. In turn, each of these depends on more specific events. Our paper describes the underlying theory, model structure, and some preliminary results intended for demonstration. In particular, we show how individual grievance evolves over time under a variety of assumptions, and how different social structures generate "echo effects" with qualitatively and quantitatively different results. This modeling effort is part of a broader project for modeling conflict in complex polities by combining computational simulations and network models.

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